Hybrid geometric-optical radiative-transfer model
for the directional reflectance of discontinuous vegetation canopies

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ABSTRACT

A new model for the bidirectional reflectance of a vegetation cover combines principles of geometric optics and radiative transfer. It relies on gap probabilities and path length distributions to model the penetration of irradiance from a parallel source and the single and multiple scattering of that irradiance in the direction of an observer. The model applies to vegetation covers of discrete plant crowns that are randomly centered both on the plane and within a layer of variable thickness above it. Crowns assume a spheroidal shape with arbitrary height to width ratio. Geometric optics easily models the irradiance that penetrates the vegetation cover directly, is scattered by the soil, and exits without further scattering by the vegetation. Within a plant crown, the probability of scattering is a negative exponential function of path length. Within-crown scattering provides the source for singly-scattered radiation, which exits with probabilities proportional to further path-length distributions in the direction of exitance (including the hotspot effect). Single scattering provides the source for double scattering, and then higher order pairs of scattering are solved successively by a convolution function. As an early exercise in validation, the model is applied to an open jack pine canopy and ground-level irradiance is predicted with good accuracy.

1. INTRODUCTION

The purpose of this paper is to present the basis of a new model for the directional reflectance of vegetation canopies and to show some initial results from attempts to validate the model using data currently available. The new model draws heavily from past work in geometric optics, and also includes multiple scattering effects in a manner similar to radiative transfer models—hence the new model's description as a hybrid geometric-optical radiative-transfer model.[1] Additionally, the model is formulated explicitly to deal with discontinuous canopies, where the presence of gaps in the canopy has significant effects on both the amount of irradiance passing directly through the canopy and the directional reflectance of the canopy—and in particular, the hotspot. Thus, gap probabilities play a major role in this new model by influencing the calculation of the distribution of pathlengths through the canopy, the distribution of single-scattering source radiation, and the calculation of an openness factor which is used to model multiple scattering and the scattering of the diffuse sky irradiance.

The scene model underlying this new hybrid model is the same discrete-object model used in our previous work on geometric optics.[2,3] The scene is composed of three-dimensional objects, which in this case are individual plant crowns, that when taken together comprise the plant canopy. Their shapes, size, and count density are all important parameters describing the scene. This scene model contrasts significantly with plane-parallel models for canopies, and is particularly suited for discontinuous plant canopies. The new hybrid model is designed for use at the scale of patches of vegetation, or areas large enough to be characterized by the means of parameters, but also homogeneous enough that those means exhibit stationarity. For forests, which have served as the primary environment driving the development of this model, the appropriate scale of application is a forest stand, which could range in area from a single hectare to tens or hundreds of hectares.

The primary focus of this paper concerns the use of the new hybrid model for studying the directional reflectance and albedo of plant canopies. However, the model also allows for calculation of a wide variety of radiation quantities as a function of height in the canopy. Thus, the model should also prove useful for studies of surface energy balance and a variety of canopy processes, such as photosynthesis and transpiration.

2. BACKGROUND

The modeling of directional reflectance of vegetated surfaces is at present a highly active research field. The research in this area is extensive and has been recently reviewed.[4-7] For the purposes of this paper, two approaches to modeling reflectance are of particular relevance: radiative transfer and geometric optics. In the radiative transfer approach, the vegetation canopy is treated as a volume-scattering medium using principles and physical approximations originally developed for the atmosphere. Usually a plane-parallel leaf canopy is assumed, which is more appropriate for crop canopies than natural vegetation canopies, which exhibit gaps and openings. Some models based on radiative transfer have included three-dimensional effects, as well as geometric-optical principles, to describe single-scattering behavior.[8]

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In the geometric-optical approach, the reflectance is modeled as a function of the self-shadowing structure of the canopy, which is treated as a collection of discrete objects—individual plant crowns—that are arranged on a plane. The pattern of sunlit and shadowed objects and background that is seen from a particular viewing position is taken as the primary factor controlling the directional reflectance.[2-3, 9-11] The pattern of light and shade associated with the scene is modeled using geometric optics, Boolean set mathematics, and theorems from stereology.

The intent of the new hybrid model is to incorporate the strengths of both approaches, which is the careful description of single scattering from geometric-optics, and the effects of higher orders of scattering from radiative transfer theory. The result of this combination should be an accurate model of the full range of quantities addressable via radiative transfer, while continuing to be applicable for discontinuous canopies and at landscape scales.

3. GAP PROBABILITIES IN VEGETATION CANOPIES

Gap probability is the probability that a photon incident upon a vegetation canopy will pass directly through the canopy without being intercepted by a leaf, branch, or stem. Typical models for gap probability are of the form

$$P_{gap} = e^{-kL/\cos\theta} \tag{1}$$

where L is the leaf area index, k is the fraction of the foliage area projected toward the angle of incidence, and θ is the zenith angle of incidence. This model has been widely used for homogeneous vegetation canopies.[12]

Most real vegetation canopies, and especially canopies of arboreal vegetation covers, depart significantly from this simple model. A significant part of the total gap will be present as gaps between individual crowns, which we have previously termed as $P(n = 0)$, where n is the number of plant crowns penetrated by a ray, and modeled as a function of crown size, shape, and count density. Besides these intercrown gaps, we explicitly modeled within-crown $P_{gap}(s)$ as a function of the within-crown pathlength s at the scale of leaves:[13]

$$P_{gap}(s,\theta) = e^{-s\,k(\theta)\,D_v} \,. \tag{2}$$

Here, $k(\theta)$ is the leaf area projection factor for the direction θ; D_v is the foliage area volume density (FAVD) with units m^{-1}. Unless otherwise mentioned, we'll assume the leaf area is uniformly distributed within the crowns, and thus D_v is constant within crowns, zero outside. We'll use $\tau(\theta) = k(\theta)D_v$ as a parameter describing projected foliage density in the direction θ. This equation is very similar to (1) in form, but it explicitly discards the assumption of a horizontal layer implied in (1). Hence it is available for dealing with various canopy structures through the distribution of pathlengths s given the geometry/structure of an average single crown. Then the mean P_{gap} over an area A becomes:

$$E(P_{gap}) = \frac{1}{A}\int\int_A e^{-\tau(\theta)\,s(x,y)}\,dx\,dy = \int_0^\infty P_{gap}(s(\theta))P(s(\theta))\,ds(\theta)\,. \tag{3}$$

Here $P(s)$ denotes probability density function of s; similarly we will use $P(y|x)$ for probability density function of y given condition x in the later text.

Note that since P_{gap} determines the proportion of radiation flux that is not scattered by foliage, $(1 - P_{gap})$ will be the fraction of the incoming flux reflected, transmitted, or absorbed by foliage. After interception, the reflected and transmitted light become scattered flux into other directions and further scattering/absorption is again determined by the exit-pathlength distribution and its correlation to the incoming-pathlength distribution. Hence the modeling of P_{gap} becomes a key linking geometric-optical and radiative-transfer models of 3-D discrete crown canopies.

A typical 1-D radiative transfer equation for a horizontally homogeneous and infinite canopy is often applied to solve for $I(z,\Omega)$, the specific energy intensity at given direction and solid angle Ω, at given wavelength in absence of polarization. Usually we can decompose I into uncollided I_0 and scattered components $I_1, I_2, \cdots$ in successive order, and apply the equation only to two successive orders of scattering iteratively, starting from attenuation only for I_0, which is:

$$I_0(x,y,z) = I_{sun}(\theta_i)\,e^{-\tau s(x,y,z,\theta_i)} = I_{sun}(\theta_i)\,P_{gap}(s(x,y,z,\theta_i))\,, \tag{4}$$

where $I_{sun}(\theta_i)$ is specific energy intensity and is assumed to be the only external radiance from zenith angle θ_i above the canopy, the canopy is assumed azimuthally isotropic and the solar azimuthal angle ϕ_i is omitted for simplicity; τ is assumed constant, and s is the within-canopy pathlength for the raybeam at the given direction to reach the point (x,y,z). Eq. (4), like Eq. (1), can be obtained from either the radiative transfer equation under given conditions or from pure statistical geometry.

In this model, we are not interested in uncollided radiance at each point, but rather in its probability distribution at the height h—that is, $P(I_0|h,\theta_i)$. Given the height, (4) tells us that I_0 at every point (x,y,z) is a monotone function of the pathlength at that point, hence the distribution of I_0 at the height h can be obtained from the probability density distribution of the

the highest positive correlation between the illumination and viewing directions. In this model we no longer assume crown surface reflection dominates. The correlation extends into the crowns, but for a given geometry, the deeper the raybeam penetrates, the weaker the correlation will be, except at the hotspot where the correlation continues to be unity along the raybeam through out the canopy. Similarly, $P(s=0|h,\theta_v)$ - $P_v(s=00|h)$ defines the distribution of surface area where negative correlation between scattering and exiting attenuation dominates. The integration of this quantity along h yields K_t in our previous G-O model, defined as areal proportion of viewed crown surface in shade.

Therefore, we model the coefficient of correlation between single scattering and exiting attenuation as:

$$\gamma(h)=\frac{\sum P_v(s=00|z)e^{(h-z)/R}}{\sum P(s=0|z,\theta_v)e^{(h-z)/R}}, \tag{9}$$

where both summations are taken for all $z \geq h$. Then the total scattering contributed to a view direction v at the height h is the sum of positively and negatively correlated means of scattering and attenuation product, weighted by the respective areal proportions.

The hotspot effect at the scale of the leaf consists of two parts. One is logically similar to the hotspot at the scale of the crown: if a raybeam reaches a point after pathlength s without collision with a leaf, there will be no leaf centered in a volume with depth equal to s and a cross-section equal to the size of a leaf. Thus, we can model the correlation function for incidence and viewing pathlengths as a function of the overlap in this volume for the incidence path with the volume of the viewing path. This is a very sharp hotspot effect for all reasonable leaf size and FAVD. Another component of contribution from leaves to directional reflection comes from the leaf angle distribution. This contribution is modeled similar to eq. (2) of Li and Strahler[3] and many other researchers' works. So we will not repeat the formulation here. Under the assumption of bi-lambertian reflectance and spherical leaf angle distribution, this integral has a simple analytic form,[16] and will contribute a mild hotspot determined only by the phase angle between i, v directions. However, if leaves have preferred orientation or non-bi-lambertian surface reflectance, this mild "hotspot" may not be at the direction backward to the sun, and digital integration may be needed. We have calculated results for such cases, but we prefer to keep the above simplified assumption unless evidence shows it to be untenable.

Then, the contribution of the single-scattering source radiance to BRDF is modeled as: $I_{1+}=I_c+I_t+I_g+I_z+I_{zz}$, where I_g and I_z are contributions from directly viewed sunlit or shaded ground; I_c is the contribution from the canopy volume in projection of K_c, I_t is from the canopy volume in projection of K_t; and I_{zz} is the contribution from scattering from the ground that is further attenuated by canopies before reaching the viewer. This contribution is assumed to be equally distributed over K_c and K_t, and thus should be part of the signatures C and T in our previous G-O model. The contribution of I_g may bear strong directional characteristics in sparse stands as we modeled before, and interested readers may refer to our earlier publications. Note that I_{zz} may also include a leaf-scale hotspot effect, but since usually in a forest the tree height is far larger than dimensions of leaves, such a narrow hotspot is practically undetectable, hence this effect is ignored in this model.

8. MULTIPLE SCATTERING AND DOWNWARD RADIATION TO THE GROUND

Knowing the distribution of single scattering source at any h, for simplicity we further assume half of the total single scattering goes upward and the other half downward at the height h in canopies:

$$J_+(h)=J_-(h)=\frac{\omega}{2}\sum P(\Delta I_0|h)\Delta I_0, \tag{10}$$

where the summation is taken for all ΔI_0 we obtained from calculating single scattering sources, and ω is the spherical albedo of leaves. On the ground, there is only upward surface scattering, so we have:

$$J_+(h=0)=\rho_s[P(n=0|h,\theta_i)\,I_{sun}+\sum P(I_0|h=0,\theta_i)I_0\,]. \tag{11}$$

Note that here J is used to indicate radiant flux with no direction considered except upward or downward, noted by +/-. J has units Watts/m^2. Part of $J_+(h)$ may directly exit the canopy through gaps between crowns. The proportion of such upward leakage is:

$$L_{k+}(h)=\frac{\Delta K_{open}(h)}{1-K_{open}(h)}, \tag{12}$$

and downward leakage will be:

$$L_{k-}(h)=\frac{\Delta K_{open}(h_1+h_2-h)}{1-K_{open}(h_1+h_2-h)}, \quad \text{for } h>h_1-R. \tag{13}$$

In these expressions,

$$K_{open}(h) = \int_0^{\pi/2} P(n = 0|h, \theta) \sin 2\theta \, d\theta \,, \tag{14}$$

and

$$\Delta K_{open}(h) = \int_0^{\pi/2} P(s = 0|h, \theta) \sin 2\theta \, d\theta \,. \tag{15}$$

At the ground, we assume $L_{k-} = 0$ and $L_{k+} = K_{open}(h = 0)$ i. e., a solid lambertian surface. This assumption can be replaced by a BRDF model of snow or soil if needed in the future. Knowing the total single scattering flux at the height h and assuming the scattering sources are spread randomly but without further leakage, the total radiation reaching another height z, $z > h$, will be:

$$J_{e+}(h-> z) = (1 - L_{k+}(h))\, J(h) \int_0^{\pi/2} \sin(2\theta)\, e^{-\tau T(z,h,\theta)} d\theta \,, \tag{16}$$

where $T(z, h, \theta) = sec\theta \int_h^z (1 - e^{-\lambda_v V_g(u)})\, du$ is the mean pathlength from h to z along a given direction. In a thin layer centered at z, the total contribution of $J(h)$ to the next order scattering will be:

$$J_s(h \to z) = \frac{\omega}{2} (1 - L_{k+}(h))\, J(h) \int_0^{\pi/2} \sin(2\theta)\, (1 - e^{-\tau\Delta s})\, e^{-\tau T(z,h,\theta)} d\theta \,, \tag{17}$$

where $\Delta s = \Delta T(z, h, \theta)$. Note the mean pathlength calculated here is different from eqs. (8)-(10), which are more accurate for small s' by taking into account the spatial correlation of pathlength segments.

The contribution of $J(h)$ to the next order scattering within a thin layer centered at h itself will be:

$$J_s(h \to h) = \frac{\omega}{2} (2 - L_{k+}(h) - L_{k-}(h))\, J(h) \int_0^{\pi/2} \sin(2\theta)\, (1 - e^{-\tau\Delta s})\, d\theta \,, \tag{18}$$

where $\Delta s = (1 - e^{-\lambda_v V_g})\, sec\theta \, \Delta h/2$.

Similarly, for any height $z < h$, we have:

$$J_{e-}(h \to z) = (1 - L_{k-}(h))\, J(h) \int_0^{\pi/2} \sin(2\theta)\, e^{-\tau T(h,z,\theta)} d\theta \,, \tag{19}$$

and

$$J_s(h \to z) = \frac{\omega}{2} (1 - L_{k-}(h))\, J(h) \int_0^{\pi/2} \sin(2\theta)\, (1 - e^{-\tau\Delta s})\, e^{-\tau T(h,z,\theta)} d\theta \,. \tag{20}$$

When $z = 0$, i. e., at the ground surface, eq. (16) no longer applies, since now we know that the other end of s' is on ground, which is always out of tree crowns. Therefore,

$$J_{e-}(h \to z = 0) = J(h) \int_0^{\pi/2} \sin(2\theta)\, E[P_{gap}|h_2 + h_1 - h, \theta]\, d\theta \,, \tag{21}$$

where $E[P_{gap}|h_2 + h_1 - h, \theta]$ is the mean of within-crown P_{gap}, quantitatively equal to the mean attenuated irradiance I_0, given an assumed unit of beam irradiance at direction θ as the only input. Then we have the contribution of $J(h)$ to the next order of scattering on the ground:

$$J_s(h \to z = 0) = \rho_s J_{e-}(h \to z = 0) \,, \tag{22}$$

where we also assume the ground is a smooth solid Lambertian surface and hence $J(h = 0)$ has no contribution to the next order scattering on the ground itself.

Similarly,

$$J_{e+}(h = 0 \to z) = J(h = 0) \int_0^{\pi/2} \sin(2\theta)\, E[P_{gap}|h_2 + h_1 - z, \theta]\, d\theta \,, \tag{23}$$

and

$$J_s(h = 0 \to z) = \frac{\omega}{2} \Delta J_{e+}(h = 0 \to z) \,. \tag{24}$$

The layer-source dilution function $J_s(h \rightarrow z)$ determines how the next order scattering source is distributed vertically given a scattering source at h, $J(h)$. A convolution-like operation then can be applied successively. Each time after such a dilution operation, we will obtain: 1) a vertical distribution $J^{(m+1)}(h)$ of scattering source for the next order; 2) an upward exiting radiance density $I_{m+}(\theta)$ for this order of scattering; 3) averaged total upward and downward radiation at any height within the canopy and the downward flux at the ground.

With the initial source distribution $J^{(1)}(h)$ generated by direct sunlight, we can obtain the next order scattering source distribution:

$$J^{(2)}(h) = \sum_{z=0}^{h_2+R} J_s(z \rightarrow h | J^{(1)}(h)) \ , \tag{25}$$

where "$|J^{(1)}(h)$" means given the vertical distribution of the first order scattering source. Similarly, any successive m'th order of scattering can be calculated from the vertical distribution of the $(m-1)$'th order scattering source. Fig. 3 shows how the first order scattering is diluted into successive orders for different canopies. The total radiation reaching any horizontal plane h will be the superposition of these orders of radiation. Therefore the accumulated scattering source at the height h will be:

$$J_{acc}(h) = \sum_{m=1}^{M} J^{(m)}(h) \ , \tag{26}$$

where M is assumed large enough so that residual scattering of higher orders is small. General speaking, the more open the canopy, the fewer orders of scattering will be needed. For all cases we have tried, $M = 16$ has been enough, even if we set ω and ρ_s to unity (no absorption at all). However, if the canopy is totally closed and absorption is low, M would need to be larger. We may develop an estimate of needed M later, but now we assume that discrete canopies have enough openness that M does not present as a problem.

Note that accumulated source $J_{acc}(h)$ is within a thin horizontal layer Δh at any height within canopies, but it may occur at a solid surface of ground. For such case, the total absorption will be:

$$J_{abs}(h=0) = J_{acc}(h=0)\ (1-\rho_s)/\rho_s \ ; \tag{27}$$

and for $h > 0$

$$J_{abs}(h) = 2\ J_{acc}(h)\ (1-\omega)/\omega \ . \tag{28}$$

The above mentioned albedo or reflectance/transmittance should be understood as being spectrally dependent. The spectral albedo above the canopy will be one minus the fraction of incoming radiation absorbed by foliage and ground. Given the incoming spectrum of illumination, the wide-band albedo above the canopy can be estimated as cited in Schaaf and Strahler.[17]

9. VALIDATION OF MODELED DOWNWELLING IRRADIANCE

A preliminary validation of directional reflectances derived from the hybrid model has already been carried out for a conifer stand near Howland, Maine.[1] In this paper, we present some early results comparing modeled downwelling irradiance values with observations collected as part of the Boreal Ecosystem-Atmosphere Study (BOREAS). For this comparison, we emphasize the multiple scattering part of the model calculation. The data were acquired by Robert Davis in a stand of old jack pine by distributing several solar pyranometers on the ground within the stand and recording downwelling solar irradiance through the daily cycle.

Fig. 4 presents superimposed traces of pyranometer readings for different locations within the stand during a three-day period. The spikes are produced by gaps that irradiate the instruments directly, but move with the sun. Beneath the spikes are baselines of irradiance that are produced by downwelling diffuse sky radiance and within-crown multiple scattering. Note that each pyranometer has its own baseline, which is determined by the degree of openness with which it views the sky. Fig. 5 shows the trace of an individual pyranometer for the three-day observation period. Superimposed on each daily trace is a smooth line derived from the hybrid model that is the sum of (1) multiple scattering generated by the single-scattering source; (2) diffuse irradiance arising from open sky area; and (3) the multiple scattering within the canopy generated by the diffuse skylight. The modeled values fit the pyranometer trace quite well, suggesting that the model describes the multiple scattering within this stand with good accuracy.

10. CONCLUDING REMARKS

In summary, the reflectance of the discrete crown canopy is characterized first by its vertical sunlit and viewed crown surface distributions and corresponding pathlength distributions. These distributions may play important roles not only in BRDF modeling, but also in many other research areas concerning forest and woodlands, such as studies of photosynthesis, soil moisture budgets, and the energy balance of snow. Then the correlation between solar direction and exiting direction is modeled at the scales of both crown and leaf. All the above are accomplished through the application of geometric optics. Then single scattering and multiple scattering are calculated according to the pathlength distributions in a way more similar to the radiative transfer approach, except that a vertical openness distribution is introduced.

Though this model may appear complicated, its final form is fairly simple and easy to apply, similar to our previous geometric-optical models. Its simplicity may yield a promising potential in future model inversion. Though the initial validation results are encouraging, more validation efforts and field measurements are needed.

11. ACKNOWLEDGEMENTS

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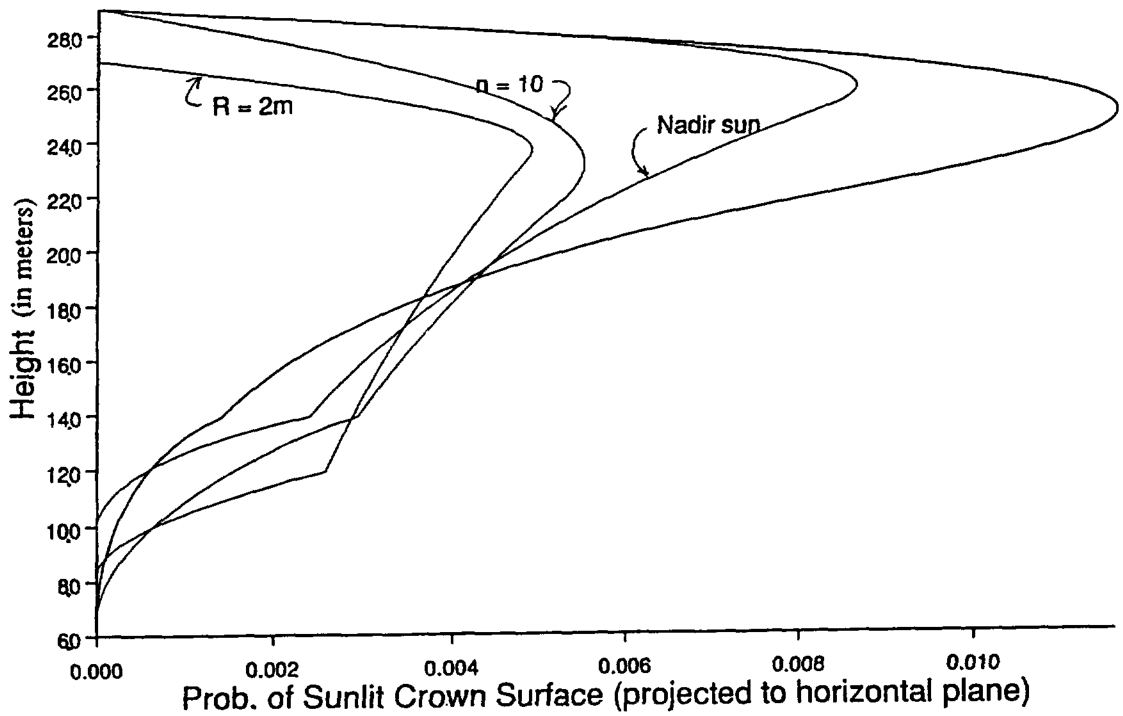

Fig. 1. Probability of sunlit crown surface. Here the default (unlabeled) curve is for a canopy composed of spherical crowns with radius $R = 4$ m that are centered randomly between heights of 10 to 25 m above the ground. Stem density is $n = 30$ per 900 m^2, providing areal coverage of 81%. The solar zenith angle is 60 degrees. The labeled curves are for the same case, with only the labeled parameter changed.

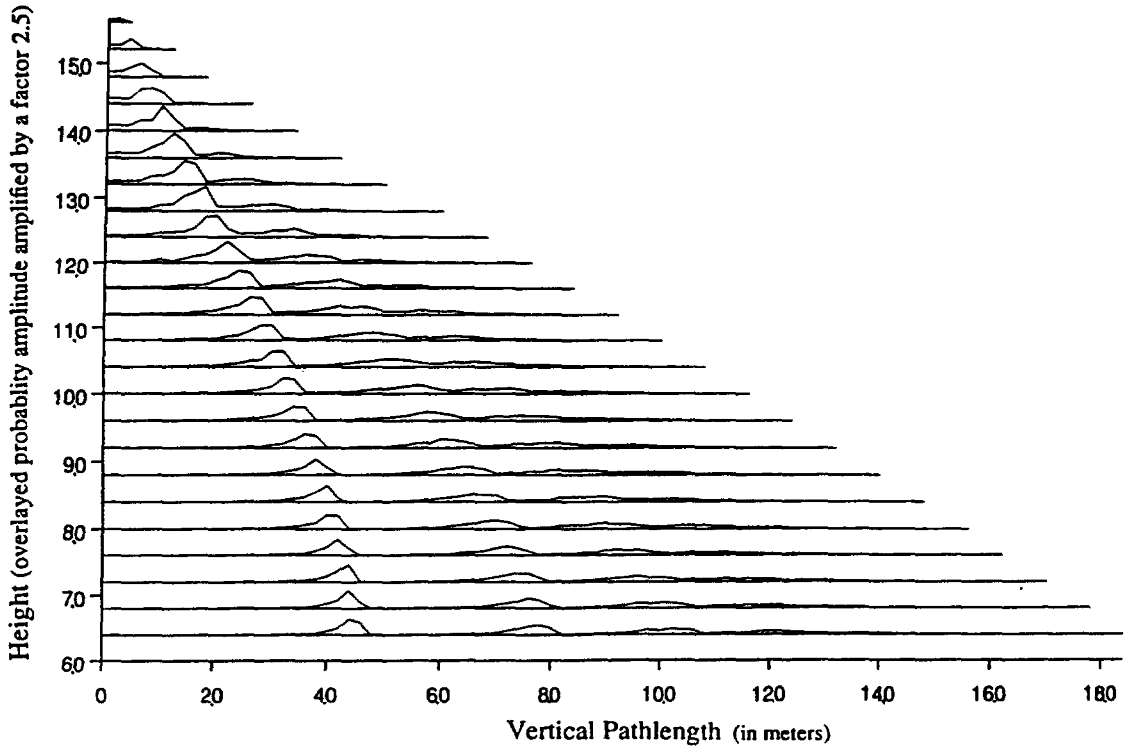

Fig. 2. Pathlength distributions at selected heights in the canopy. Each curve is a probability density function (scale not shown). The X-axis for each curve is placed at the proper canopy height and the probability is shown by the curve above it with amplitude expanded by a factor of 25. All parameters are the same as the default case in Fig. 1 except that the crown centers are randomly centered between 10 m to 12 m above the ground.

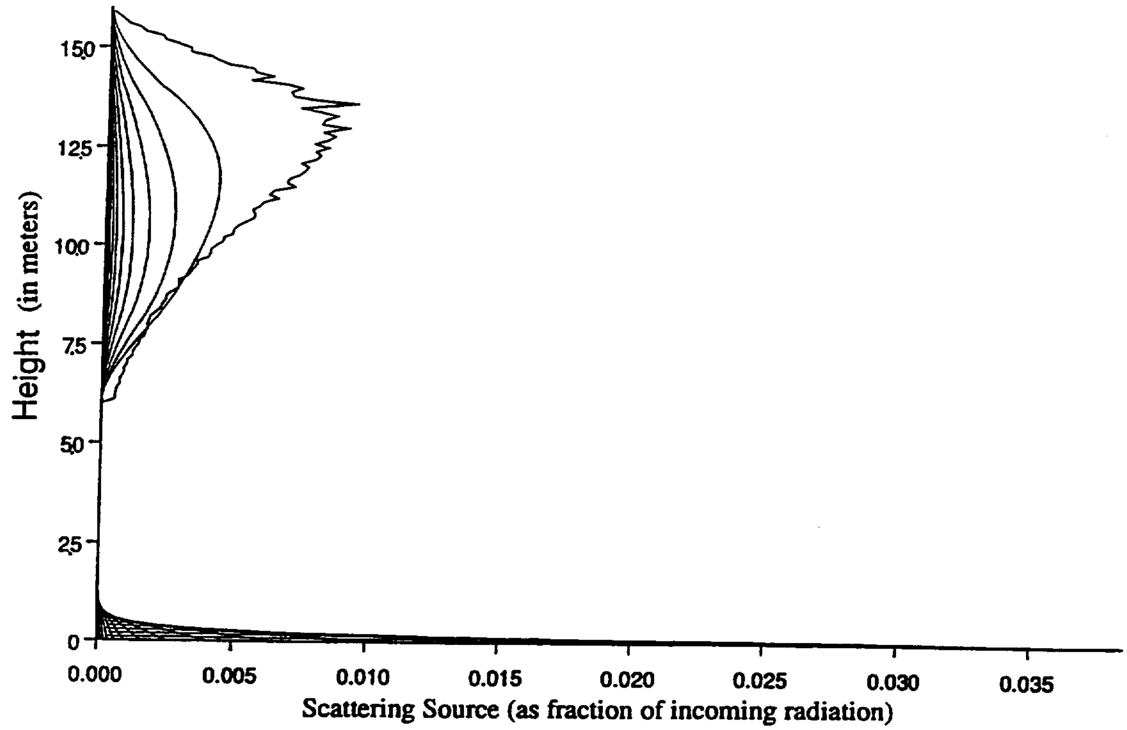

Fig. 3. Dilution of multiple scattering sources within canopy. Successive curves are for successive orders of scattering with the lowest orders to the right.

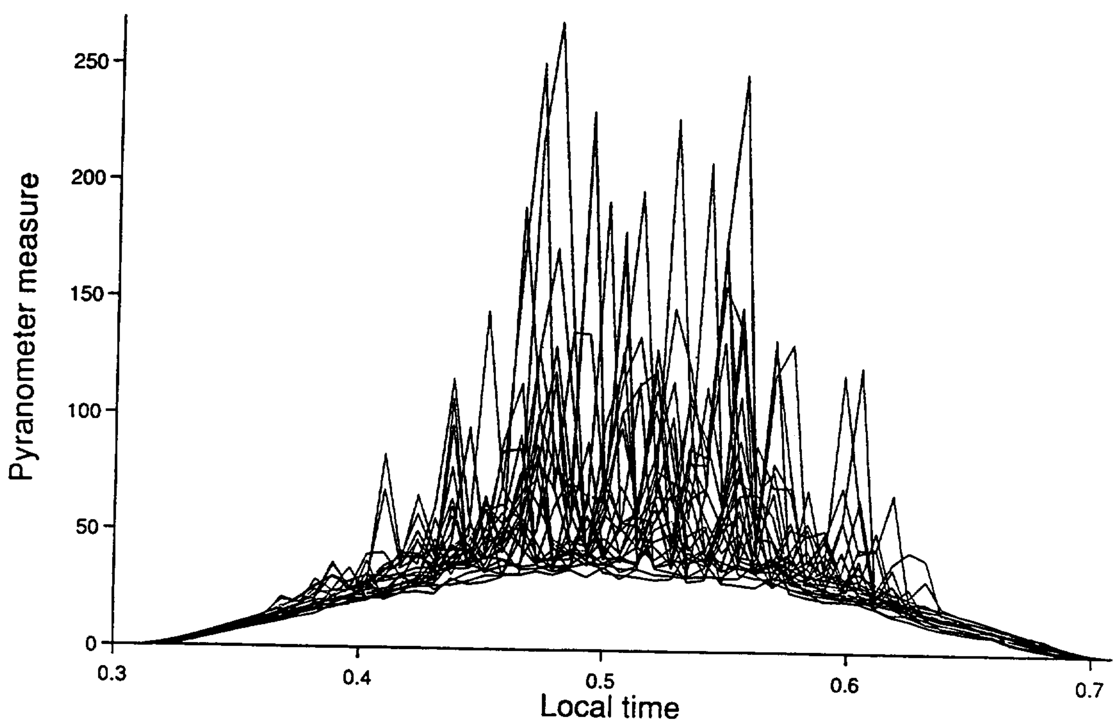

Fig. 4. Pyranometer measurements, old jack pine stand. Continuous traces of several pyranometers placed on the ground at different locations in the stand. The traces for a three-day period are normalized and superimposed. Solar zenith angle is 69°.

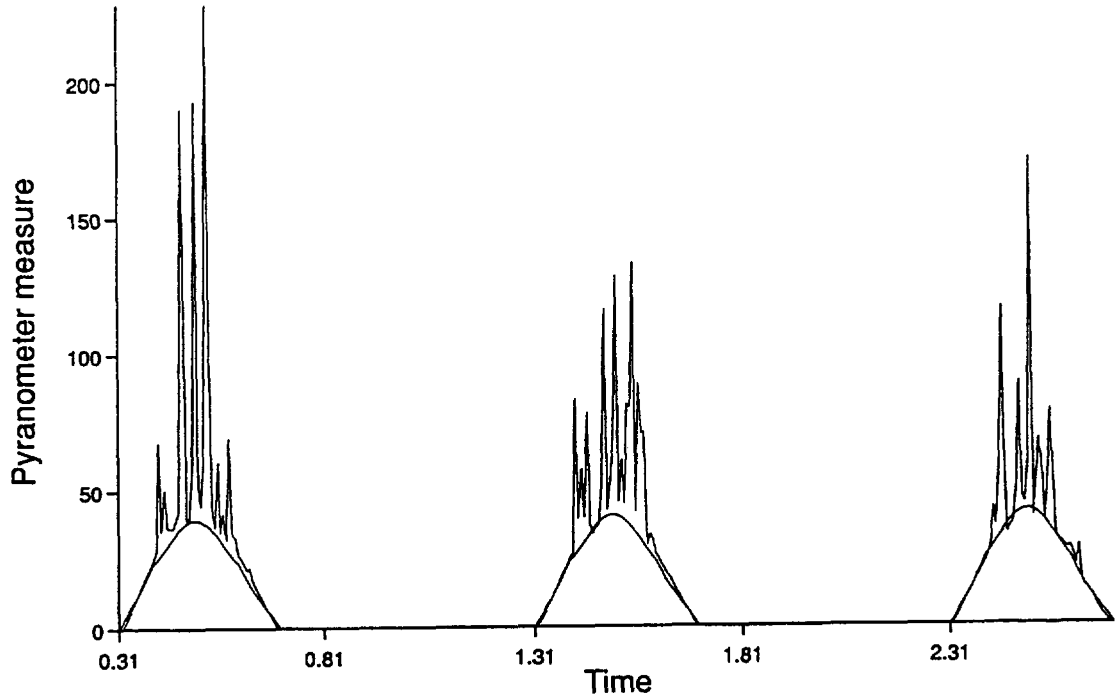

Fig. 5. Single pyranometer traces during a three-day period. Superimposed on the jagged pyranometer traces are smooth modeled values for the diffuse irradiance, including skylight and light that is multiply scattered by the canopy layer.